JEAN LUDWIG: WALKING WITH A MATHEMATICIAN

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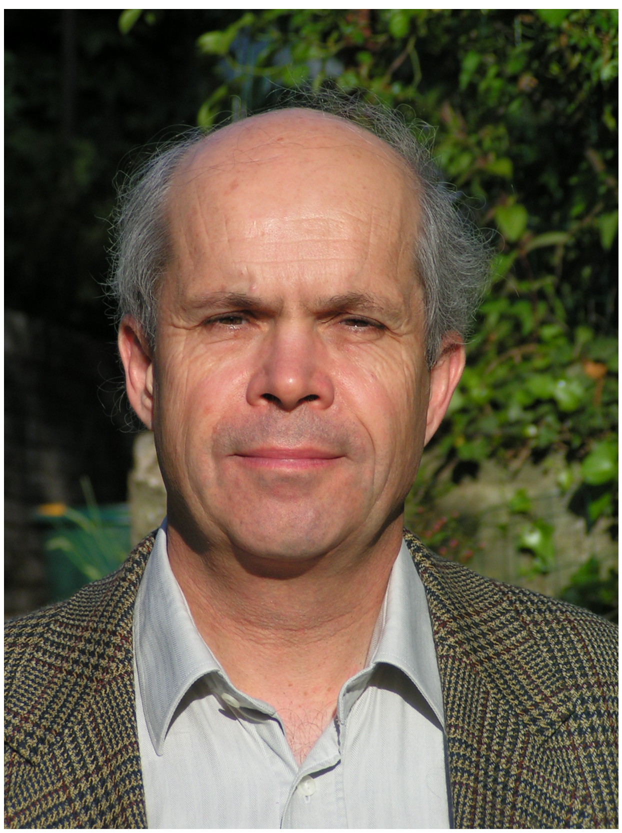

Jean Ludwig was born on June 21, 1947 in Dudelange, a small city from Luxembourg, exactly at the border with France. To continue his advanced studies, he came to Heidelberg, one of the most famous and old University in Germany, with the firm project to study Astronomy. He changed right away his mind and chose Mathematics. He spent four years at the University of Heidelberg as a student and got the Diplommathematiker in 1972. Right after, Jean left Heidelberg and had a long experience until specialized himself recent universities. First he came to Bielefeld, in North Westphalia, following his thesis adviser, Horst Leptin. Bielefeld was then a pretty recent university, since it was created in 1969, three years before the coming of Jean.

The Bielefeld University created in '69. Jean arrived in '72.

In this fine place, Jean got his PHD in 1976 and became an assistant professor until 86. As usual in German Universities, the university of Bielefed was equipped with large and comfortable buildings, a very good library, and so on...

After many short term positions in Nancy, Luxembourg and Metz, he was nominated as a full Professor at the University of Paul Verlaine- Metz in 1990. This still was a recent university, since it was founded in 70.

The Metz University created in '70. Jean arrived in '90.

Metz is very close to Luxembourg, and to the German border. Moreover it was a German city during more than 40 years, so any one can imagine that there were only minor differences between Metz and Bielefeld. Jean immediately faced some difficulties: for instance, to present his application, he needed an official French translation of his diploma; but the result was awful: Jean's accreditation to supervise was translated as research an ability to teach mathematics for pupils. Also, Jean found himself in a small math department with a very poor library, housed in a very few rooms just below the roof of the sciences faculty. Indeed, at this time, the staff did not have enough chairs to be all seated together!

But the major difficulty was the complete and absolute lack of harmonic analysis in Metz at that moment, though there were some good groups working in harmonic analysis in Nancy and Strasbourg. Some of our colleagues were therefore thinking that the future of the Metz department could only be in developing applied Mathematics.

After Jean's arrival in Metz, the harmonic analysis group grew rapidly to be become the largest and the most active group in mathematics at Metz. Some years later, Jean became director of the messian '*Unité mixte de Recherche en Mathématiques*', which organized there, with the support of the French National Center for Scientific Research, all the mathematical activities. At the moment where, for typically French reasons, this unité de recherche was amalgamated with the larger research unit in Nancy, harmonic analysis was more developed and represented in Metz than in Nancy.

Jean Ludwig supervised the scientific works of many active researchers in the field of non-commutative harmonic analysis, six of them from Tunisia. The list of his PhD students is as follows:

- Dhieb Sami. (1991-1995) . PHD: March 1995.
- Baklouti Ali.(1992-1995) PHD: June 1995.
- Andele Joseph. (1993-1997) PHD: June 1997.
- C. Molitor - Braun. (1993-1996) PHD: May 1996.
- P. Mabele. (1994-1998) PHD: June 1998.
- Mint El Hacin. (1995-1999) PHD: June 1999.
- Alexander David. (1996-2000) PHD: June 2000.
- Jahwar Abdennader. (2000-2004) PHD June: 2004.
- Laurent Scuto. (2001-2005) PHD: October 2005.
- Elloumi Mounir. (2006-2009) PHD: June 2009.
- Lahiani Raza. (2006-2010) PHD: March 2010.
- Regeiba Hedi. (2010-2014) PHD: June 2014.
- Günther Janne-Kathrin. (2012-2016) PHD: September 2016.

One of the first Jean's PHD students was Ali Baklouti. Thus, very quickly, Jean started naturally a strong collaboration with the university of Sfax.

The Sfax faculty of Sciences created in ‘87. Jean arrived in ‘92.

It was at this time a very young university: the faculty of Sciences was founded in 87. The beginning of this collaboration was completely similar to the starting of Jean's activity in Metz. First he had to explain to a young Tunisian immigration officer: “Yes, the Grand Duché de Luxembourg exists: it is a real European country”. Then Jean met the very small math department, housed just below the roof, and temporarily installed in the Ecole Nationale d'Ingénieur de Sfax. Jean, with the immediate and very efficient help of Ali Baklouti, developped there harmonic analysis, as a completely new thematic.

As in Metz, and under the impulsion and direction of Ali Baklouti, the harmonic analysis group of Sfax became very important. It is now a well known laboratory, with a very large and well established reputation, for instance on study of solvable homogeneous space, with many fine active people, among the main protagonist of the scientific life of Tunisia.

Then, very naturally, Jean and Ali directed some CMCU cooperation contracts for four years, between Metz, Dijon and Clermont-Ferrand on one side, Sfax and Monastir on the other side. This contract was really successful: the present meeting is a clear evidence of its success: many people here were related through this contract, and they continue to cooperate now.

The mathematical research activities of Jean was endorsed through many projects from the "Centre Universitaire de Luxembourg" and then from 2003, by the completely new University of Luxembourg. In a main part thanks to the efforts of Jean and Carine Molitor, good mathematical teaching and research has now become a reality in Luxembourg.

The Luxembourg University created in 2003. Jean arrived in 1986.

Jean was effectively a walking Mathematician, with a strong influence all along his way, in many centres. His principal motivation for all these journeys was of course his many various collaborations. All his students and collaborators went to Metz or Luxembourg to see him, discuss with him, write common articles, and so on...

Jean Ludwig's research profile is quite various and includes harmonic analysis on locally compact groups, unitary and Banach space representations of solvable Lie groups and semi-simple Lie groups, topology of the dual space of exponential solvable Lie groups, invariant differential operators on Lie groups, Banach and Fréchet algebras associated with groups. He wrote many impressive documents in Mathematics, including two research books.

The following is just a very small selection among the more than Jean's 90 papers, related to different aspects of his work. This will certainly explain the word "Mathematician" in our title. To be an efficient and productive mathematician is clearly a characteristic of Jean Ludwig.

First, Jean is one of the best specialists in the study of the topology of the unitary dual of an exponential group. Indeed, his notion of variable groups was the first main tool in the proof of the continuity of the inverse of the Kirillov mapping. This notion was found at the very beginning of the Jean's career. Here is a short selection of his papers about this topological question:

- *On the behaviour of sequences in the dual of a nilpotent Lie group.*
  Math. Ann. 287 (1990), no. 2, 239-257:
  Let G be a simply connected nilpotent Lie group. The unitary dual $\hat{G}$ of G is homeomorphic to the space $\mathfrak{g}^\star/G$ of the G-coadjoint orbits. The topology of

these spaces is of course not Hausdorff, an explicit description of this topology is essentially given by the description of the limit sets L of sequence (π_n) of representations associated to orbits with same dimension. In this paper, Jean introduces the fundamental notion of variable groups in order to compute explicitly the strength of the convergence of a subsequence, *i. e.* the natural numbers I_σ such that, for each f in the Pedersen ideal associated to the class of orbits, $\lim_k \operatorname{tr} \pi_{n_k}(f) = \sum_{\sigma\in L} I_\sigma \operatorname{tr} \sigma(f)$.

- H. Leptin and J. Ludwig, *Unitary representation theory of exponential Lie groups.*
  Expositions in Mathematics, 18. de Gruyter & Co., Berlin, 1994:
  Probably the most natural setting for the description of unitary duals by the orbit method is the class of exponential groups. As in the nilpotent case, for an exponential group G, the Kirillov map K is a bijective map between the space $\mathfrak{g}^\star/G$ of coadjoint orbits and the unitary dual of G. This book is the first complete and explicit presentation of this theory. In this work, Horst leptin and Jean present the class of exponential group, prove clearly and explicitly that K is a bijection, then they expose the notion of variable groups and show how to use it in the study of the bi-continuity of K. They finally can describe the range of $L^1(G)$ through any irreducible unitary representation. In our opinion this book is with the monograph of Bernat and all. one of the most important presentation work of these theory.

- *Dual topology of diamond groups.*
  J. Reine Angew. Math. 467 (1995), 67-87:
  Looking now for a natural solvable non exponential group, the diamond group G, to prove the continuity of the map K, Jean needs to rely convergences of sequence of (essentially generic) orbits with convergences of representations (π_n). In this paper Jean built explicit elements u in the universal enveloping algebra of $\mathfrak{g}$, allowing him to compute the limit of orbits through the knowledge of the $\pi_n(u)$.

But this knowledge of limits of sequences in unitary duals recently allowed also Jean to completely characterize the $C^\star$ algebra of some Lie groups:

- *Limit sets and strengths of convergence for sequences in the duals of thread-like Lie groups.* (with R. J. Archbold, and G. Schlichting)
  Math. Z. 255 (2007), no. 2, 245-282.
  Even for thread-like nilpotent Lie group, the $C^\star$ algebra was not known. In this paper jean with his co-authors describe completely both the limit sets of sequence of representations (or orbits) and the strength of the corresponding convergences. Roughly speaking, the part of the Fourier transform of these

$C^\star$ algebras corresponding to stable subset in $\mathfrak{g}^\star$, with diffeormorphic orbits are simple to describe, moreover if (π_n) is a sequence of such representations, and L its limit set, the relation : $\lim_n \|\pi_n(f)\| = \sup_{\sigma\in L} \|\sigma(f)\|$ holds. In this paper, the authors prove that this condition completely characterizes the $C^\star$ algebra of the considered groups.

- *An isomorphism between group C^*-algebras of $ax + b$-like groups.* (with Lin, Ying-Fen)

  Bull. Lond. Math. Soc. 45 (2013), no. 2, 257-267.

  In this fine paper, the authors study the $C^\star$ algebras of different semi-direct products on the form $G_\mu = \mathbb{R} \rtimes_\mu \mathbb{R}^d$ associated to a diagonalizable action μ of $\mathbb{R}$ on $\mathbb{R}^d$. Especially they consider distinct actions μ, μ' such that $C^\star(G_\mu)$ and $C^\star(G_{\mu'})$ are isomorphic, and they give a necessary and sufficient condition for this isomorphism.

- *C^*-algebras with norm controlled dual limits and nilpotent Lie groups.* (with H. Regeiba)

  J. Lie Theory 25 (2015), no. 3, 613-655.

  This paper gives the (very difficult) proof that the very natural description of $C^\star$ algebra of thread-like groups extends to any nilpotent group with dimension at most 5.

Let us now illustrate the long and fruitful collaboration of Jean Ludwig with Ali Baklouti just by one of their first work on the explicit description and decomposition of the restriction to a nilpotent subgroup of a monomial representation:

- *Désintégration des représentations monomiales des groupes de Lie nilpotents.* (with A. B.)

  J. Lie Theory 9 (1999), no. 1, 157-191.

  This work deals with the question of the explicit description of an intertwining operator between a monomial representation $\pi = \mathrm{Ind}_H^G \chi$ of a nilpotent group G, induced by a character χ of an analytic subgroup H. In fact π is disintegrated into a direct integral of representations σ associated to coadjoint orbits in $(\ell + \mathfrak{h}^\perp)/H$, if $i\ell|_\mathfrak{h} = d\chi$. The authors describe here an explicit realization of an intertwining operator between π and the direct integral of the σ.

- *The Penney-Fujiwara Plancherel formula for nilpotent Lie groups.* (with A. B.)

  J. Math. Kyoto Univ. 40 (2000), no. 1, 1-11.

  Using the preceding results, the Penney formula associated to a monomial representation $\pi = \mathrm{Ind}_H^G \chi$ (G is simply connected nilpotent, H is a connected closed subgroup of G) is established in a very precise and explicit form. If the decomposition of the restriction τ of π to H contains only finite multiplicities, the space of $D_\tau(G/H)$ is characterized: in this case, it is isomorphic to $\mathbb{C}(f+\mathfrak{h}^\perp)^H$, if if is the differential of χ.

- *La formule de Penney-Plancherel des restrictions à multiplicités finies des groupes de Lie nilpotents.* (with A. B., and H. Fujiwara)
  Adv. Pure Appl. Math. 4 (2013), no. 1, 21-40.
  Let G be a simply connected nilpotent Lie group, K an analytic subgroup and $\pi \in \hat{G}$. AThe authors give the explicit Plancherel and Penney-Plancherel formulae for the decomposition of the restriction of π to K, in the case where the multiplicities in this decomposition are all finite. They compute an explicit intertwining operator for this decomposition, by diagonalizing this operator, they can prove that the algebra of differential operators preserving the smooth vectors for π and commuting with the K-action is commutative.

Similarly, let us recall the common work of Jean Ludwig and Didier Arnal on the moment set of a unitary representation:

- *La convexité de l'application moment d'un groupe de Lie.* (with D. A.)
  J. Funct. Anal. 105 (1992), no. 2, 256-300.
  For a nilpotent group, the natural moment set associated to an irreducible unitary representation, viewed as a symplectic action on the corrseponding projective Hilbert space is the closed convex hull of the corresponding coadjoint orbit. In this paper, it is essentially proved that this result extends for solvable groups.

- *Separation of unitary representations of connected Lie groups by their moment sets.* (with L. Abdelmoula, D. A. and M. Selmi)
  J. Funct. Anal. 228 (2005), no. 1, 189-206.
  The usual moment set of a representation does not characterizes this representation. By using analytic vectors, Jean proves in this article that the natural extension of this moment set to the universal enveloping algebra of any simply connected Lie group G characterizes the irreducible representations of G.

But of course, the most constant and frequent Jean's co-author is Carine Molitor-Braun. Let us recall this long and various collaboration through the thematic of study of some natural group algebras, like:

- *Weighted group algebras on groups of polynomial growth.* (with G. Fendler, K. Gröchenig, M. Leinert, and C. Molitor-Braun)
  Math. Z. 245 (2003), no. 4, 791-821.

This paper does not concern solvable or nilpotent Lie groups. if G is any locally compact group, a weight ω on G is a real Borel function such that $\omega(s) \geq 1$, $\omega(st) \leq \omega(s)\omega(t)$ and $\omega(s^{-1}) = \omega(s)$ $(s,\ t \in G)$. The space $L^1(G,\omega)$ is then a natural Banach algebra for the convolution. The group G is of polynomial growth if there is a symmetric compact neighbourhood U of 1 such that $G = \cup_n U^n$ and $|U^n| \leq Cn^d$ for some d and any n. G is of strict polynomial growth if $C_1 n^d \leq |U^n| \leq C_2 n^d$ for some positive C_i. Put $v_U^\omega(k) = sup\{\omega(y)|\ y \in U^{|k|}\}$ $(k \in \mathbb{Z})$. Suppose G of polynomial growth and $\lim_{k\to\infty} v_U^\omega(k)^{1/k} = 1$, then $L^1(G,\omega)$ is symmetric. Conversely if G is of strict polynomial growth, ω tempered, and $L^1(G,\omega)$ symmetric, then $\lim_{k\to\infty} v_U^\omega(k)^{1/k} = 1$.

- *Fine disintegration of the left regular representation.* (with C. Molitor-Braun) J. Algebra Appl. 4 (2005), no. 6, 683-706.

  This paper describe completely and explicitly the decomposition of the action of the group algebra $L^1(G)$ acting on $L^2(G)$ by left convolution: Jean and Carine prove that $L^2(G)$ splits into a direct sum of isotopic components of a family of left-invariant differential operators. On each components, the restriction of the regular representation is essentially multiplicity free.

- *Spectral synthesis for flat orbits in the dual space of weighted group algebras of nilpotent Lie groups.* (with C. Molitor-Braun and D. Poguntke) Trans. Amer. Math. Soc. 365 (2013), no. 8, 4433-4473.

  Consider a nilpotent, simply connected Lie group G, extend a irreducible representation $\pi \in \hat{G}$ to the Banach algebra $L^1_\omega(G)$, where ω is a symmetric polynomial weight. This article gives the description of the set of all two sided closed ideals I related to $\ker\pi$, if the coadjoint orbit associated to π is flat: $\mathcal{O}_\pi = \ell + \mathfrak{g}(\ell)^\perp$.

The preceding is a very small (more or less arbitrary) aspect of the very various mathematical interests of Jean. It is well known that he is always ready to help you with his expertise, let us illustrate this point by the small example of the solvability of differential operators:

- *Sub-Laplacians of holomorphic L^p-type on rank one AN-groups and related solvable groups.* (with D. Müller) J. Funct. Anal. 170 (2000), no. 2, 366-427.

  Let G be an exponential group, consider a point ℓ in the dual $\mathfrak{g}^\star$ of the Lie algebra $\mathfrak{g}$ of G. Suppose that $\mathfrak{g} = \mathfrak{g}(\ell) + [\mathfrak{g},\mathfrak{g}]$, and ℓ is not vanishing on the intersection of the lower central series in $\mathfrak{g}$. Then a sub-Laplacian L is of holomorphic L^1-type: there is λ in the spectrum of L such that, if m is a function such that $m(L)$ is bounded on L^1, then $m(L)$ can be extended holomorphically to a complex neigborhood of λ. If there is an open subset of such ℓ in $\mathfrak{g}^\star$, the result holds for each L^p, $1 < p < \infty$, $p \neq 2$.

- *Uniqueness of solutions to Schrödinger equations on 2-step nilpotent Lie groups.* (with D. Müller)
  Proc. Amer. Math. Soc. 142 (2014), no. 6, 2101-2118.
  Consider a two-step nilpotent group with Lie algebra $\mathfrak{g}$, fix a basis (V_i) of $[\mathfrak{g}, \mathfrak{g}]$, and a real symmetric matrix $[a_{ij}]$, and the Shrödinger equation
  $$\partial_t u(t,g) = i \sum_{i,j} a_{ij} V_i V_j u(t,g).$$
  Jean and D. Müller give a sufficient condition on $f_0(g) = u(0,g)$ and $f_T(g) = u(T,g)$ $(T > 0)$ such that the only solution is $u = 0$.

Finally, it is worth mentioning the long collaboration of Jean with Hidenori Fujiwara, and we will just illustrate it by their recent book:

- H. Fujiwara and J. Ludwig, *Harmonic analysis on exponential solvable Lie groups.*
  Springer Monographs in Mathematics. Springer, Tokyo, 2015.
  As it is said in the review in MathScinet of this book : "The appearance of this book is an important event which will strongly influence the development of the area". In fact this recent book gives a presentation of the essential tools in the theory of representations of exponential groups, but also describes many important results in harmonic analysis, like
  - The proof of the commutativity conjecture: for exponential group, the decomposition of a monomial representation contains only finite multiplicity if and only if the algebra of invariant differential operators on the section of the corresponding lin bundle is commutative,
  - The proof of the restriction conjecture: suppose now G nilpotent, then the restriction of an irreducible representation π has only finite multiplicities if and only if, in the quotient of the universal enveloping algebra of G by the kernel of π, the algebra of K-invariant elements is commutative.

Jean Ludwig occupied many administrative positions during his career. In Octobre 2014, Jean was nominated as Professor emeritus, researcher at the Laboratoire Institut Elie Cartan, University of Lorraine. Being one of the prominent researchers in his field, he is still interested in the orbit method for solvable Lie groups, where many interesting problems remain to be solved.

Walking through Jean's life, let us mention finally some real, concrete walks: Jean and us, we were very frequently walking together, speaking and chatting. We were speaking essentially of the future, our own future, the future of our families, our institutions, and our countries... We were sure to be able to modify, ameliorate, even create this future.