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Abstract : This thesis is situated in the field of algebraic combinatorics and deals with the construction of several combinatorial and algebraic structures on the species of finite topological spaces \mathbb{T} .

After defining a pre-Lie product (grafting law) on the species of connected finite topological spaces \mathbb{V} , we show that this underlying pre-Lie structure defines a coproduct Δ_{\searrow} on \mathbb{T} different from those already defined by F. Fauvet, L. Foissy and D. Manchon in 2017, then we show the connection between the Grossman-Larson product of this graft law and the coproduct Δ_{\searrow} .

We prove that the species of connected finite posets \mathbb{U} (i.e., the species of connected T_0 finite topological spaces), is a free pre-Lie algebra and is a cofree NAP coalgebra, then we give an explicit duality between this product and this proposed coproduct. Then we show that these results hold on \mathbb{V} .

We define two different doubling species \mathbb{D} and $\widetilde{\mathbb{D}}$ of the species \mathbb{T} . We also define a product and a coproduct on \mathbb{D} and $\widetilde{\mathbb{D}}$ respectively. Then we prove that there exist graded bimonoid structures on \mathbb{D}_X and $\widetilde{\mathbb{D}}_X$. Then we build a twisted pre-Lie structure $\circ\rightarrow$ on a doubling \mathbb{W} of the species \mathbb{V} of connected finite topological spaces. Then we prove that \mathbb{W} is a left module on this twisted pre-Lie algebra. Then we show that there is a "derivation" relation linking this structure $\circ\rightarrow$ to the pre-Lie structure on the left denoted \searrow obtained on \mathbb{V} .

Finally we describe the topological quandles of cardinal three and four. Then we propose open questions around the structures of Hopf algebra on the species of finite topological quandles.

keywords : finite topological spaces, Species, bialgebra, twisted bialgebra, doubling bialgebra, Hopf algebra, quandle, topological quandle, rooted tree, operad, pre-Lie algebra, NAP algebra.