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Abstract : In the first chapter, we recall some background on the topology of the unitary dual of a locally compact group. Then we summarize some basic facts about the structure of compact Lie groups and their representations. We conclude this chapter by briefly explaining the orbit method for Lie groups with co-compact nilradical.

In chapter two, we consider the Cartan motion group G_0 associated to a given compact Riemannian symetric pair (G, K). Under some assumptions on the pair (G, K), we give a precise description of the set $(\widehat{G_0})_{gen}$ of all equivalence classes of generic irreducible unitary representations of G_0. We also determine the topology of the space $(\mathfrak{g}_0^\ddagger/G_0)_{gen}$ of generic admissible coadjoint orbits of G_0 and we show that the bijection between $(\widehat{G_0})_{gen}$ and $(\mathfrak{g}_0^\ddagger/G_0)_{gen}$ is a homeomorphism. Furthermore, in the case where the pair (G, K) has rank one, we prove that the unitary dual $\widehat{G_0}$ is homeomorphic to the space $\mathfrak{g}_0^\ddagger/G_0$ of all admissible coadjoint orbits of G_0.

Chapter three deals with the dual topology of the semi-direct product $G_n = \mathbb{T}^n \ltimes \mathbb{H}_n$, where $\mathbb{H}_n$ is the $(2n + 1)$-dimensional Heisenberg group, and $\mathbb{T}^n$ is the n-dimensional torus acting on $\mathbb{H}_n$ by automorphisms. In this chapter, we describe the space of admissible coadjoint orbits of the group G_n and we determine the topology of this space. We show that the bijection between the unitary dual $\widehat{G_n}$ of G_n and its admissible coadjoint orbit space is a homeomorphism.

The fourth chapter is a note in wich we show that every irreducible unitary representation of the Euclidean motion group $M_n = SO(n) \ltimes \mathbb{R}^n$, $n \geq 2$, is characterized by a particular element in its generalized moment set.

This thesis is ended by an appendix in which we explain some useful results about compact Riemannian symmetric pairs.(Collaboration with Majdi Ben Halima).